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Hedlund's theorem for compact laminations by hyperbolic surfaces

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If $\mathcal{L}$ is a compact minimal lamination by surfaces of negative curvature, we give a sufficient condition for the horocycle flow on its unit tangent bundle to be minimal. The geodesic and horocycle flows over compact hyperbolic surfaces have been studied in great detail since the pioneering work in the 1930's by E. Hopf and G. Hedlund. Such flows are particular instances of flows on homogeneous spaces induced by one-parameter subgroups, namely, if G is a Lie group, K a closed subgroup and N a one-parameter subgroup of G, then N acts on the homogeneous space $K\backslash G$ by right multiplication on left cosets. One very important case is when $G = SL(n, \mathbb{R})$, $K = SL(n, \mathbb{Z})$ and N is an unipotent one parameter subgroup of $SL(n, \mathbb{R})$, i.e., all elements of N consists of matrices having all eigenvalues equal to one. In this case $SL(n, \mathbb{Z})\backslash SL(n, \mathbb{R})$ is the space of unimodular lattices. By a theorem by Marina Ratner , which gives a positive answer to the Raghunathan conjecture, the closure of the orbit under the unipotent flow of a point $x \in SL(n, \mathbb{Z})\backslash SL(n, \mathbb{R})$ is the orbit of x under the action of a closed subgroup $H(x)$. This particular case already has very important applications to number theory, for instance, it was used by G. Margulis and Dani and Margulis to give a positive answer to the Oppenheim conjecture. When $n = 2$ and Γ is a discrete subgroup of $SL(2, \mathbb{R})$ such that $M := \Gamma\backslash SL(2, \mathbb{R})$ is of finite Haar volume, and N is any unipotent one-parameter subgroup acting on M, Hedlund proved that any orbit of the flow is either a periodic orbit or dense. When Γ is cocompact the flow induced by N has every orbit dense, so it is a minimal flow. The horocycle flow on a compact hyperbolic surface is a homogeneous flow of the previous type and most of the important dynamic, geometric and ergodic features are already present in this 3-dimensional case.

On the other hand, the study of Riemann surface laminations has recently played an important role in holomorphic dynamics polygonal tilings of the Euclidean or hyperbolic plane , moduli spaces of Riemann surfaces, etc. It is natural then to consider compact laminations by surfaces with a Riemannian metric of negative curvature and consider the positive and negative horocycle flows on the unit tangent bundle of the lamination. In this paper we give a condition that guarantees that both these flows are minimal if the lamination is minimal.

joint work with Matilde Martínez