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On a peculiar conformally defined class of surfaces and foliations

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In [LW1], the authors proposed to study the extrinsic conformal geometry of foliations on 3-manifolds of constant sectional curvatures. By *extrinsic conformal geometry*, we mean such geometric properties which (1) can be expressed in terms of the second fundamental form (of the leaves) and (2) are invariant under Möbius transformations.

The simplest property of this sort is *umbilicity*: a surface (in particular, a leaf) L is umbilical whenever the two principal curvatures k_1 and k_2 of L are equal. In [LW1], the authors proved that umbilical foliations (that is, foliations by umbilical leaves) on compact manifolds of non-zero constant curvature do not exist.

Since then, we (working, in different configurations, together with Adam Bartoszek, Gilbert Hector and Rémi Langevin), obtained a number of negative results concerning existence of foliations by leaves enjoying several geometric properties. For example, we proved that (nonsingular!) foliations by Dupin cyclides [LW1] and surfaces with constant local conformal invariants [BW] (see [CSW] for the definitions) do not exist on compact manifolds of constant non-zero curvature and that foliations by canal surfaces (that is envelopes of one-parameter families of spheres) do not exist on closed hyperbolic manifolds [HLW].

Positive results of this sort have been obtained as well: on the sphere S^3, nonsingular foliations by canal surfaces (and by *special canal surfaces* defined and studied in [BLW]) and singular foliations by Dupin cyclides do exist and have been classified ([LW2] and [LS]).

In this talk, we will define and discuss a new (?) class $\mathcal{S}$ of surfaces: those built of pieces of canals and pieces of spheres. Roughly speaking, a surface L belongs to $\mathcal{S}$ whenever one of its principal conformal curvatures (again, see [CSW] for a definition) vanishes at all the non-umbilical points. We will show that, from the topological point of view, any "reasonable" surface can be represented by a surface of our class $\mathcal{S}$ and that there exist many foliations of 3-manifolds by the leaves of this class. In particular, we shall show that

(1) all the surfaces listed as *generic leaves* in either [CC] or [Gh] can be represented by elements of this class,

(2) several closed 3-manifolds (like T^3, S^3 and some others) admit

foliations by surfaces of class $\mathcal{S}$.

It seems that our class $\mathcal{S}$ should be interesting not only from the point of view of pure mathematics (geometry) but also for computer aided geometric design (CAGD) [1].

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